卐 Chapter 8 Elastic and Inelastic Collision

Syllabus: 8.1 Collision of particles; collision in laboratory and center of mass system, cross section

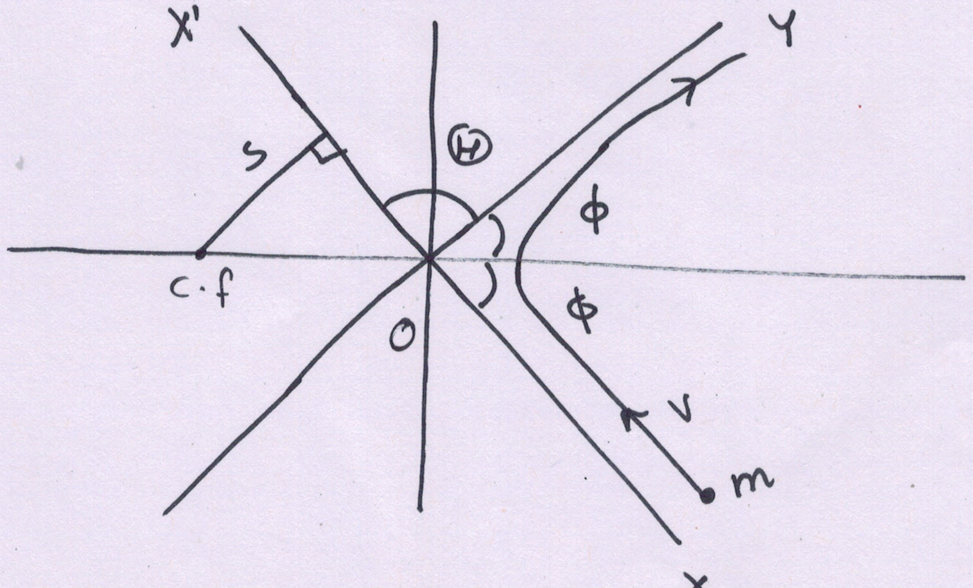
8.2. Rutherford scattering

Scattering:

The change in direction of motion of particle due to interaction with another particle is known as scattering.

The angle between initial and final direction of motion of particle is called scattering angle.

Consider a particle of mass (m) is projected towards the center of force (c.f.) from infinity along XO with velocity $(\vec{v})$. As it approaches the point A, the point of closest approach to the center of force, it begins to move away from the center of force and finally picks up the straight line path OY as shown in fig.



The angle $X'OY = \Theta$ is called angle of scattering & perpendicular distance (s) is called the impact parameter.

The angle of scattering is given by,

$$\Theta = \pi - 2\phi$$

where, ϕ is an angle made by initial and final direction with symmetry axis.

The KE and angular momentum is given by,

$$KE\,(E) = \frac{1}{2} m v^2$$

$$\&\quad (l) = mvs = s\sqrt{2mE}$$

where (s) is called impact parameter.

Again, at turning point;

$$r = r_{min} \quad \text{and} \quad \dot{r} = 0$$

So, Total energy,

$$(E) = KE + PE$$

$$(E) = \frac{1}{2} m v^2 + V(r)$$

$$(E) = \frac{1}{2} \frac{l^2}{m r_{min}^2} + V(r_{min})$$

Let the repulsive force is inverse square in nature;

i.e. $f(r) = \frac{k}{r^2} \Rightarrow V(r) = -\frac{k}{r}$

Then,

$$E = \frac{l^2}{2m\, r_{min}^2} - \frac{k}{r_{min}} \quad \text{—— ①}$$

The equation of conic section is,

$$r = \frac{p}{1+\epsilon\cos\theta}$$

If $\theta = 0°$, $\cos\theta = 1$

$$\therefore \quad r_{min} = \frac{p}{1+\epsilon}$$

Putting value of r_{min} in eqn ①.

$$E = \frac{l^2(1+\epsilon)^2}{2m\,p^2} - \frac{k(1+\epsilon)}{p}$$

$$E = \frac{l^2(1+\epsilon)^2 m^2 k^2}{2m\, l^4} - \frac{k(1+\epsilon)\cdot mk}{l^2} \qquad \left(\because p = \frac{l^2}{mk}\right)$$

$$E = \frac{mk^2(1+\epsilon)^2}{2l^2} - \frac{mk^2(1+\epsilon)}{l^2}$$

$$E = \frac{mk^2}{2l^2}\left[1+\cancel{2\epsilon}+\epsilon^2 - 2 - \cancel{2\epsilon}\right]$$

$$E = \frac{mk^2}{2l^2}(\epsilon^2 - 1)$$

$$\epsilon^2 = 1 + \frac{2El^2}{mk^2}$$

$$\epsilon = \sqrt{1 + \frac{2El^2}{mk^2}}$$

For $E > 0$; $\epsilon > 1$ i.e. the conic section is hyperbola.

Repulsive

For hyperbolic asymptote:

$$\tan\phi = \pm\frac{b}{a}$$

$$\tan^2\phi = \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \epsilon^2 - 1 = \frac{2El^2}{mk^2} \qquad \because \frac{b^2}{a^2} = \epsilon^2 - 1$$

$$\tan\phi = \sqrt{\frac{2El^2}{mk^2}}$$

We have,

$$\Theta = \pi - 2\phi$$

or, $$\frac{\Theta}{2} = \frac{\pi}{2} - \phi$$

or, $$\tan\frac{\Theta}{2} = \tan\left(\frac{\pi}{2} - \phi\right) = \cot\phi$$

or, $$\tan\frac{\Theta}{2} = \frac{1}{\tan\phi}$$

or, $\tan\frac{\Theta}{2} = \sqrt{\frac{mk^2}{2El^2}}$ $\quad \because l = s\sqrt{2mE}$

or, $\tan\frac{\Theta}{2} = \sqrt{\frac{mk^2}{2E\cdot s^2\, 2mE}}$

or, $\tan\frac{\Theta}{2} = \sqrt{\frac{k^2}{4E^2 s^2}}$

or, $\tan\frac{\Theta}{2} = \frac{k}{2sE}$

or, $$\boxed{\Theta = 2\tan^{-1}\left(\frac{k}{2sE}\right)}$$

This is expression for scattering angle.

Rutherford Scattering Cross-Section

Rutherford scattering is the scattering of charged particle by coulomb's potential of the form $V(r) = -\frac{k}{r}$

where, $k = ZZ'e^2$ where, Ze is charge of target particle.

The expression for scattering angle,

$$\Theta = 2\tan^{-1}\left(\frac{k}{2sE}\right)$$

or, $\tan\frac{\Theta}{2} = \frac{k}{2sE}$

or, $s = \frac{k}{2E}\left(\tan\frac{\Theta}{2}\right)^{-1} = \frac{k}{2E}\cot\frac{\Theta}{2}$

Now differentiating w.r.to Θ

$$\frac{ds}{d\Theta} = \frac{k}{2E}\left(-\operatorname{cosec}^2\frac{\Theta}{2}\right)\frac{1}{2}$$

Now, differential scattering cross section is,

$$\sigma(\Theta) = \frac{s}{\sin\Theta}\left|\frac{ds}{d\Theta}\right|$$

$$\sigma\,\Theta = \frac{k}{2E}\cot\frac{\Theta}{2}\cdot\frac{1}{\sin\Theta}\;\frac{K}{2E}\cdot\operatorname{cosec}^2\frac{\Theta}{2}\cdot\frac{1}{2}$$

$$\sigma\,\Theta = \left(\frac{k}{2E}\right)^2\cdot\frac{1}{2}\;\frac{\cos\frac{\Theta}{2}}{\sin\frac{\Theta}{2}}\cdot\frac{1}{2\sin\frac{\Theta}{2}\cos\frac{\Theta}{2}}\;\operatorname{cosec}^2\frac{\Theta}{2}$$

$$\sigma(\Theta) = \frac{1}{4}\left(\frac{k}{2E}\right)^2\operatorname{cosec}^4\frac{\Theta}{2}$$

$$\boxed{\sigma(\Theta) = \frac{1}{4}\left(\frac{zz'e^2}{2E}\right)^2\operatorname{cosec}^4\frac{\Theta}{2}}$$

This is the expression for Rutherford scattering cross-section. This relation shows that the differential scattering cross-section ~~is~~ of α-particle by a nucleus is inversly proportional to the square of energy of the incident particle.

Differential Scattering cross-section

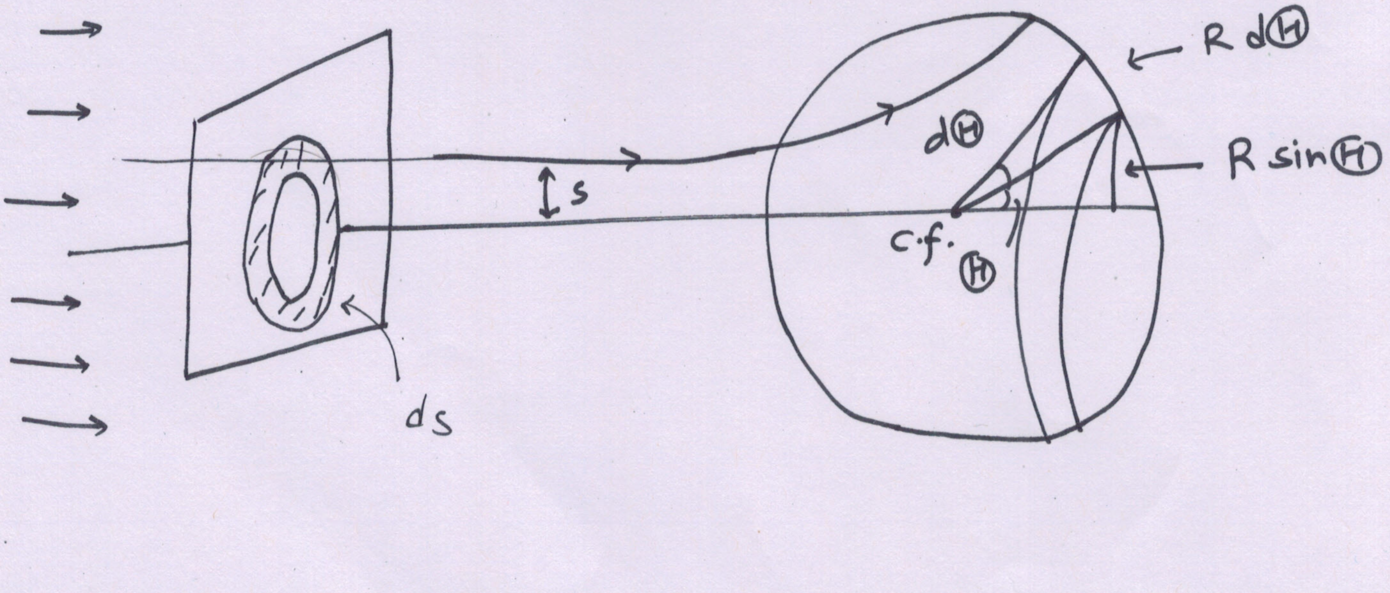
Consider the beam of particles having equal energy is incident on the center of force (c.f.).

The differential scattering cross-section is denoted by $\sigma(\Theta)$ and is given by,

$$\sigma(\Theta) = \frac{\text{No. of particles scattered per unit solid angle per sec}}{\text{Incident intensity } (I)}$$

where, intensity is defined as number of particles incident on the unit area perpendicular to the direction of the beam per sec.

Here, $\sigma(\Theta)$ has the dimension of area.



from fig,

Area of the surface forming solid angle at the center of force, $= \text{Circumference} \times \text{thickness}$

$$= 2\pi R \sin\Theta \times R\, d\Theta$$

$$= 2\pi R^2 \sin\Theta\, d\Theta$$

Now, solid angle $(d\Omega) = \dfrac{\text{Area}}{R^2}$

$$(d\Omega) = 2\pi \sin\Theta\, d\Theta$$

Number of particles scattered per unit solid angle per sec $= \sigma(\Theta) \cdot I$

Number of particles scattered ~~per~~ through solid angle $(d\Omega)$ per sec $= \sigma(\Theta)\, I \cdot d\Omega$

$$= \sigma(\Theta)\, I \cdot 2\pi \sin\Theta\, d\Theta$$

This is equal to the number of particles passing through the circular strip of area $2\pi s\, ds$. where (s) be impact parameter.

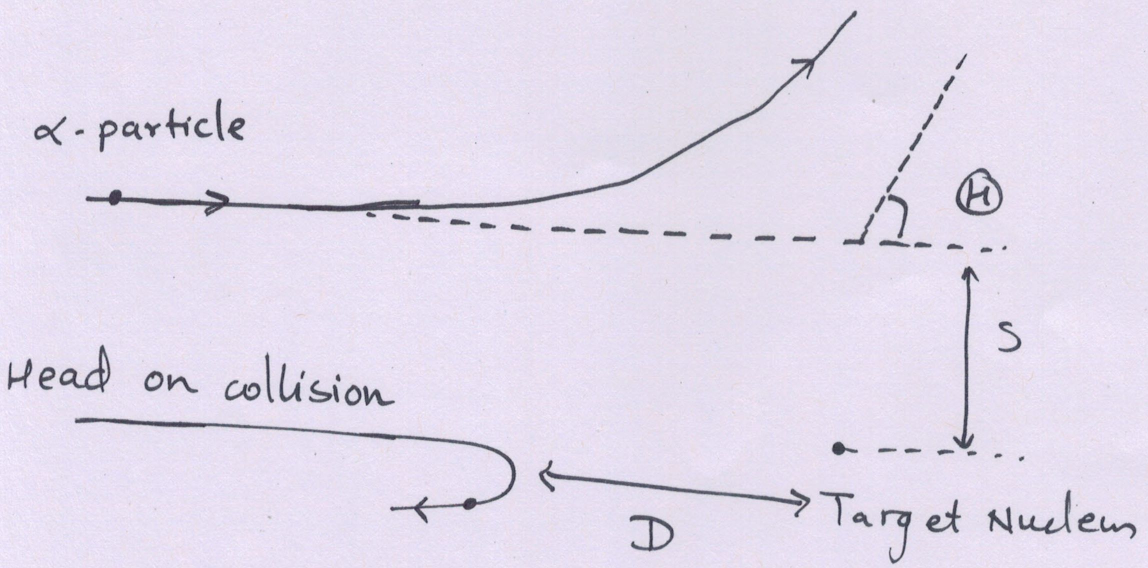
Here, we considered the particles passing within impact parameter (s) and $(s + ds)$ is,

$$I \cdot 2\pi s\, ds = \sigma(\Theta)\, I \cdot 2\pi \sin\Theta\, d\Theta$$

$$\Rightarrow \quad \sigma(\Theta) = \frac{s}{\sin\Theta}\left|\frac{ds}{d\Theta}\right|$$

Scattering cross-section formula is the measure of probability of number of particles scattered per unit solid angle per unit time in the direction specified by scattering angle Θ.

In other words scattering cross-section is the effective area provided by the scatterer to the incident particle for scattering purpose.



Impact Parameter (s) → The minimum distance to which the α-particle would approach the nucleus if there were no forces between them. for head on collision $s=0$.

Distance of closest approach (D) :- The minimum distance to which the α-particle in head on collision would approach the nucleus.