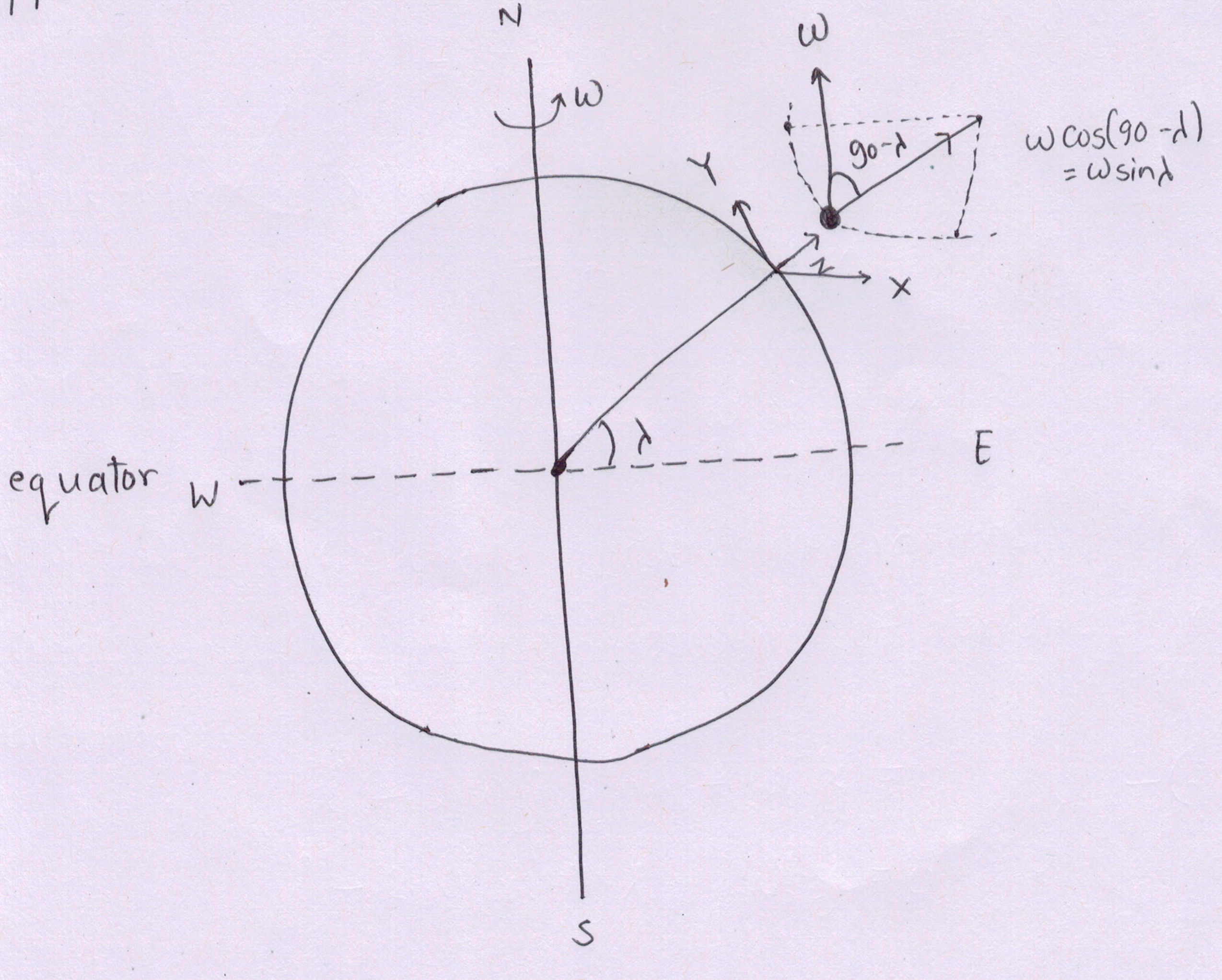
卐 Foucault's Pendulum

Foucault performed his pendulum experiment in Paris which demonstrates the rotation of earth.

The pendulum was introduced in 1851 and was the first experiment to give direct evidence of earth's rotation.

He took a pendulum with a heavy iron bob (28 kg) suspended by means of long steel wire (67 m long) the upper end of which was attached to rigid support.



At an ~~a~~ latitude λ, the angular velocity(ω) can be resolved into two components $\omega \sin\lambda$ and horizontal component $\omega \cos\lambda$ in N-S direction.

Here, $\omega \cos\lambda$ will not have any appreciable effect on the pendulum. The vertical component ~~ω~~ will make the plane of oscillation to rotate with angular velocity $\omega \sin\lambda$.

Now, time of one rotation of the plane of oscillation will be given by,

$$T = \frac{2\pi}{\omega \sin\lambda}$$

When the pendulum moves along a horizontal line, the Coriolis force acts in the perpendicular direction to it's velocity and hence plane of oscillation turns.

The rotation of the plane of oscillation of Foucault's pendulum demonstrates the fact that earth rotates about it's axis.

The Faucault's pendulum is suspended along the vertical Z-axis. The bob of pendulum vibrates with small amplitude in the horizontal XY plane.

The equation of motion for simple pendulum is,

$$\ddot{r} = -k^2 r \qquad \text{—— (1)}$$

where, $k^2 = \frac{g}{l}$; $g \rightarrow$ acceleration due to gravity

$l \rightarrow$ effective length of pendulum

In presence of coriolis force.

$$\ddot{r} = -k^2 r + 2(\vec{v} \times \vec{\omega})$$

Let,

$$\vec{v} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k}$$

$$\vec{\omega} = \omega_x\,\hat{i} + \omega_y\,\hat{j} + \omega_z\,\hat{k}$$

$$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

$$\therefore \vec{v} \times \vec{\omega} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dot{x} & \dot{y} & \dot{z} \\ \omega_x & \omega_y & \omega_z \end{vmatrix}$$

As the pendulum oscillates about Z-axis so X- and Y- components can be written as;

$$\ddot{x} = -k^2 x + 2(\dot{y}\,\omega_z - \dot{z}\,\omega_y)$$

$$\ddot{y} = -k^2 y - 2(\dot{z}\,\omega_x - \dot{x}\,\omega_z)$$

But $\dot{z} = v_z = 0$ i.e. the velocity along Z-axis is zero.

Then above eq^{n}s becomes;

$$\ddot{x} = -k^2 x + 2\dot{y}\,\omega_z \quad \text{— (2)}$$

$$\ddot{y} = -k^2 y + 2\dot{x}\,\omega_z \quad \text{— (3)}$$

Multiplying (3) by i & adding with (2).

$$\ddot{x} + i\ddot{y} = -k^2(x + i\dot{y}) + 2\dot{y}\,\omega_z + 2i\dot{x}\,\omega_z$$

$$= -k^2(x + iy) + 2\dot{y}\,\omega_z - \frac{2\dot{x}\,\omega_z}{i}$$

$$\ddot{x} + i\ddot{y} = -k^2(x + iy) - 2i\omega_z(\dot{x} + i\dot{y}) \quad \text{— (4)}$$

Let $u = x + iy$ then eq^n (4) becomes,

$$\ddot{u} = -k^2 u - 2i\,\omega_z\,\dot{u}$$

$$\ddot{u} + 2i\omega_z \dot{u} + K^2 u = 0 \quad -⑤$$

This is second order differential equation and it's solution can be written as;

$$u = e^{-i\omega_z t}\left[A e^{iK't} + B e^{-iK't}\right] \quad -⑥$$

where, $K' = K^2 + \omega_z^2 \simeq K^2$ (in absence of coriolis force $\omega_z = 0$)

$$K' = K$$

so,

$$u_0 = A e^{ikt} + B e^{-ikt}$$

Now eq^n ⑥ can be written as,

$$u = u_0 e^{-i\omega_z t}$$

$$(x + iy) = (x_0 + iy_0) e^{-i\omega_z t}$$

This is the equation of plane wave where velocity is $-\omega_z$. This shows that the plane of oscillation of pendulum oscillates with angular velocity ω_z.

At any latitude, $\omega_z = \omega \sin\lambda$

Time period, $(T) = \dfrac{2\pi}{\omega \sin\lambda}$

At pole: $\lambda = 90^\circ$

$$T = \frac{2\pi}{\omega \sin 90} = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{2\pi}{T}\right)} = T = 24\,\text{hrs}$$

At equator ~~T~~: $\lambda = 0^\circ$

$$T = \frac{2\pi}{\omega \sin 0^\circ} = \frac{2\pi}{0} = \infty$$

From above discussions, it is clear that earth has it's rotational motion about a fixed axis.