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We have,

$$\vec{a} = \vec{a'} + 2\vec{\omega} \times \frac{d'\vec{r}}{dt} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \frac{d\vec{\omega}}{dt} \times \vec{r}$$

The additional terms appear because of relative ~~motion~~ rotation of two co-ordinate systems.

The third term $\vec{\omega} \times (\vec{\omega} \times \vec{r})$ is the centripetal acceleration.

$$\left| \vec{\omega} \times (\vec{\omega} \times \vec{r}) \right| = \omega^2 r \sin\theta$$

The second term is called coriolis acceleration.

The last term is the acceleration produced by the angular velocity of primed system with respect to unprimed system when $\vec{\omega}$ is not constant.

If we assume that Newton's law of motion holds true in unprimed system:

$$m\vec{a} = \vec{F}$$

$$m\vec{a'} + m\,\vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2m\vec{\omega} \times \frac{d'\vec{r}}{dt} + m\frac{d\vec{\omega}}{dt} \times \vec{r} = \vec{F}$$

Then, Newton's law in primed system takes the form:

$$m\vec{a'} = \vec{F} - \left[m\,\vec{\omega}\times(\vec{\omega}\times\vec{r}) + 2m\,\vec{\omega}\times\frac{d'\vec{r}}{dt} + m\,\frac{d\vec{\omega}}{dt}\times\vec{r} \right]$$

Let us put, $m\vec{a'} = F_{eff}$

$$\frac{d'\vec{r}}{dt} = v'$$

or, $$\vec{F_{eff}} = \vec{F} - 2m(\vec{\omega}\times\vec{v'}) - m(\vec{\omega}\times(\vec{\omega}\times\vec{r})) - m\frac{d\vec{\omega}}{dt}\times\vec{r}$$

Here, $2m(\vec{\omega}\times\vec{v'})$ is called Coriolis force ~~which~~ which is perpendicular to the plane containing $\vec{\omega}$ & $\vec{v'}$.

For uniformly rotating system $\vec{\omega}$ is constant so $\frac{d\vec{\omega}}{dt} = 0$.

Hence, $$\vec{F_{eff}} = \vec{F} - 2m(\vec{\omega}\times\vec{v'}) - m\,\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{a'} = \frac{\vec{F}}{m} - 2(\vec{\omega}\times\vec{v'}) - \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

when the force $\vec{F}$ is only due to earth's attraction only then,

$$\frac{\vec{F}}{m} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) = g - \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{g}_e$$

Here we have combined the effect of the gravitational force and centrifugal force terms because both are indistinguishable from eachother.

Thus, $m\vec{g}_e = m\left[g - \vec{\omega} \times (\vec{\omega} \times \vec{r})\right]$

The magnitude of $m\left(\vec{\omega} \times (\vec{\omega} \times \vec{r})\right) = m\omega^2 r \sin\theta$

The maximum value of $\omega^2 r$ is $3.38 \, cms^{-2}$ (at equator $\theta = 90°$) which is 0.3% of acceleration due to gravity.

i.e. $\vec{\omega} \times (\vec{\omega} \times \vec{r}) << g$

So we can neglect $\vec{\omega} \times (\vec{\omega} \times \vec{r})$ terms.

i.e. $\vec{g}_e = g$

Hence, $\boxed{\vec{a}' = \vec{g}_e - 2(\vec{\omega} \times \vec{v}')}$

Q: what do you mean by Coriolis Force?

⇒ The fictitious force $-2m(\vec{\omega} \times \vec{v'})$ which appears when the particle is ~~is~~ in motion relative to the rotating frame of reference is called Coriolis force.

Coriolis force is directly proportional to the angular velocity of rotating frame of reference and velocity of particle in S' frame.

It is zero if either ω is zero or v' is zero (particle is at rest in S' frame)

The deflection of freely falling object towards east in the northen hemisphere is perfect example of Coriolis force.

Coriolis force leads to formation of cyclone, trade winds, bending of rivers etc.