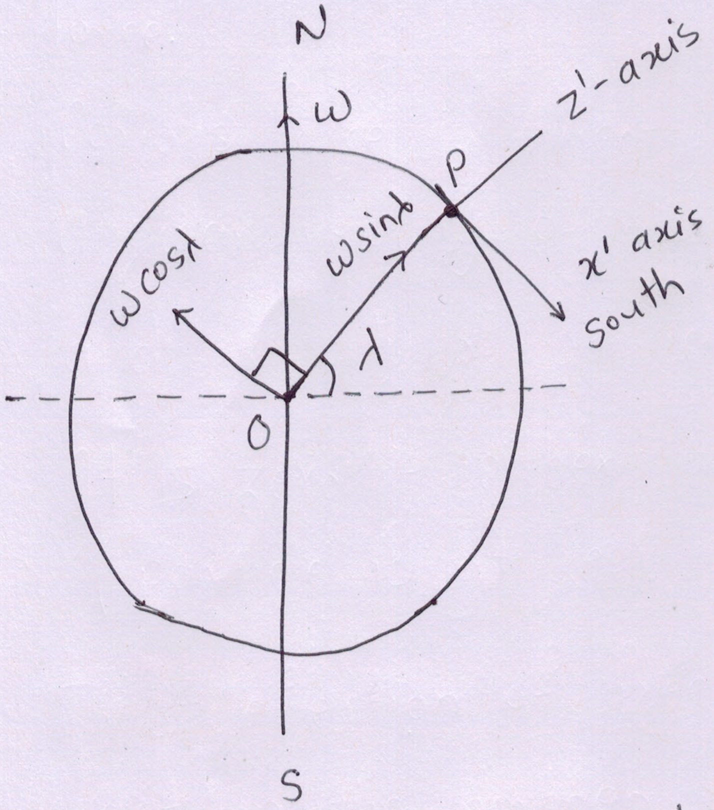
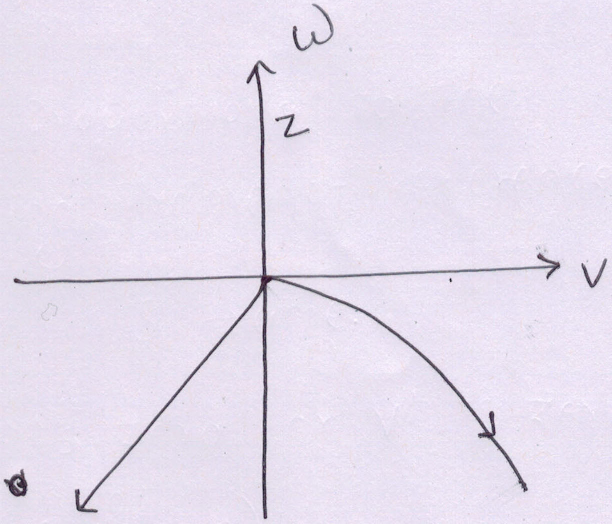
Free fall of a body on earth's surface

Let us take the problem of free fall of a body on earth's surface. The earth constitutes a rotating system with angular velocity 0.7×10^{-4} rad s^{-1}.





Let us make observations at a latitude of λ.
The earth rotates about N-S axis with angular velocity (ω). Let z'- axis be ~~the~~ taken in the direction OP, x'- axis points towards south and y'- axis points towards east.

Since particle moves towards the earth, it's velocity in rotating system (v') is not zero.

The acceleration of the body is given by,

$$\vec{a'} = \vec{g}_e - 2(\vec{\omega} \times \vec{v'})$$

or, the equation of motion of the body is,

$$m\vec{a'} = m\vec{g}_e - 2m(\vec{\omega} \times \vec{v'}).$$

Let, $\vec{v'} = \hat{i}' \dot{x}' + \hat{j}' \dot{y}' + \hat{k}' \dot{z}'$

$\vec{\omega} = \hat{i}' \omega_x' + \hat{j}' \omega_y' + \hat{k}' \omega_z'$

From fig. $\omega_x' = -\omega \cos\lambda$

$\omega_y' = 0$

$\omega_z' = \omega \sin\lambda$

Therefore,

$$m\vec{a'} = m\vec{g}_e - 2m\left[(\hat{i}' \omega_x' + \hat{j}' \omega_y' + \hat{k}' \omega_z') \times (\hat{i}' \dot{x}' + \hat{j}' \dot{y}' + \hat{k}' \dot{z}')\right]$$

$$m\vec{a'} = m\vec{g}_e - 2m\left[\hat{k}' \omega_x' \dot{y}' - \hat{j}' \omega_x' \dot{z}' + \hat{j}' \omega_z' \dot{x}' - \hat{i}' \omega_z' \dot{y}'\right]$$

$$m\vec{a} = m\vec{g}_e - 2m\left[-\hat{i}' \omega_z' \dot{y}' + \hat{j}'(\omega_z' \dot{x}' - \omega_x' \dot{z}') + \hat{k}' \omega_x' \dot{y}'\right]$$

And the equations which govern the free fall are:

$$m \frac{d^2 x'}{dt^2} = 2m\, \omega_z'\, \dot{y}' \qquad (X\text{- component})$$

$$\text{or,} \quad m \frac{d^2 x'}{dt^2} = 2m\omega \sin\lambda \frac{dy'}{dt}$$

Y-component,

$$m \frac{d^2 y'}{dt^2} = -2m \left(\omega_z'\, \dot{x}' - \omega_x'\, \dot{z}'\right)$$

$$m \frac{d^2 y'}{dt^2} = -2m\omega \sin\lambda \frac{dx'}{dt} \overset{-}{\cancel{+}} 2m\omega \cos\lambda \frac{dz'}{dt}$$

Z- component,

$$m \frac{d^2 z'}{dt^2} = -mg_e - 2m\, \omega_x'\, \dot{y}'$$

$$m \frac{d^2 z'}{dt^2} = -mg_e + 2m\omega \cos\lambda \frac{dy'}{dt}$$

we have taken $(-mg_e)$ because direction of $\vec{g}$ are z' are in opposite direction.

The body falls under the action of gravity so the velocity will be almost constant along z'-axis. There may be a small deviation from this direction, giving a negligible value of $\frac{dy'}{dt}$ and $\frac{dx'}{dt}$. ~~i.e.~~ Neglecting $\frac{dy'}{dt}$ and $\frac{dx'}{dt}$ we find that;

$$m \frac{d^2x'}{dt^2} = 0 \qquad \text{—(1)}$$

$$\cancel{m\, d^2y'} \quad m \frac{d^2y'}{dt^2} = -2m\omega \cos\lambda \frac{dz'}{dt} \qquad \text{—(2)}$$

$$\& \qquad m \frac{d^2z'}{dt^2} = -mg_e \qquad \text{—(3)}$$

The first equation gives $\frac{dx'}{dt}$ = constant i.e. there is no deviation in the north-south direction (since positive x-axis points towards south)

From eqn (3) we get, $\frac{dz}{dt} = -g_e t$

Putting this value in eqn (2),

$$\frac{d^2y'}{dt^2} = 2\omega g_e t \cos\lambda$$

Integrating twice and taking,

$$\left(\frac{dy'}{dt}\right)_{t=0} = 0 \quad \& \quad (y')_{at\ t=0} = 0$$

we get,

$$y' = \frac{1}{3} \omega g_e t^3 \cos\lambda$$

If (h) be the height of free fall, the time spent in the flight is,

$$t = \sqrt{\frac{2h}{g_e}}$$

Suppose the body falls from 100 m. above the ground, then,

$$t = \sqrt{\frac{2\times100}{9.8}} = \ldots$$

$$\omega = 0.7\times10^{-4}\ \text{rad}\,s^{-1}.$$

Therefore, the displacement at the equator is, $(\lambda=0)$ is,

$$y' = \frac{1}{3} \omega g_e t^3$$

$$\approx 3\ \text{cm}$$

which is difficult to be detected since positive y'-axis points towards east, hence the body deflects by 3 cm towards east or $+y'$ ~~ax~~ direction which is perpendicular to both ω & v'.

consequently this displacement is due to Coriolis force. (which is perpendicular to both $\vec{\omega}$ & $\vec{v'}$).